



THE POSSIBILITIES OF THE LEONTIEF MODEL BASED ON MATRIX THEORY IN THE ANALYSIS OF MODERN ECONOMIC PROCESSES

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***Abstract.** This article examines the analytical possibilities of the Leontief input–output model formulated within the framework of matrix theory for the analysis of modern economic processes. The study focuses on the mathematical representation of intersectoral economic relationships using systems of linear equations and matrix operations. Special attention is given to the role of the technical coefficient matrix and the Leontief inverse in determining equilibrium conditions and assessing the direct and indirect effects of changes in final demand. The paper highlights the importance of eigenvalue analysis and stability conditions in evaluating the feasibility and sustainability of economic systems. Furthermore, the applicability of matrix-based extensions of the Leontief model to contemporary economic challenges, including global value chains, regional development, and environmental–economic interactions, is discussed. The results demonstrate that matrix theory significantly enhances the explanatory and analytical power of the Leontief model, making it a valuable tool for modern economic analysis and policy evaluation.*

***Keywords.** Leontief model, matrix theory, input–output analysis, technical coefficient matrix, Leontief inverse, intersectoral balance, economic modeling, equilibrium analysis.*

Introduction. Modern economic systems are characterized by increasing structural complexity, intensive intersectoral interactions, and rapid transformations driven by globalization, technological progress, and digitalization. These features significantly complicate the processes of economic analysis, forecasting, and strategic planning, thereby necessitating the application of advanced mathematical methods capable of capturing the internal mechanisms of economic systems. In this context, matrix-based economic models play a crucial role in providing a systematic and quantitative description of intersectoral relationships.

One of the most influential and widely used approaches in this field is the input–output model developed by Wassily Leontief. The Leontief model offers a rigorous mathematical framework for analyzing the flow of goods and services between economic sectors by representing production technologies and resource dependencies in matrix form. By employing tools from matrix theory, such as inverse matrices, eigenvalues, and stability conditions, the model enables a detailed examination of equilibrium states and structural interdependencies within an economy.

The theoretical foundation of the Leontief model is closely linked to linear algebra, where the system of intersectoral balance equations is expressed as a matrix equation relating total output, intermediate consumption, and final demand. This formulation allows researchers to assess how changes in final demand propagate throughout the entire economic system, affecting production levels across multiple sectors. The concept of the Leontief inverse, in particular, provides valuable insights

into multiplier effects and the overall responsiveness of the economy to external shocks.

In recent years, the applicability of the Leontief model has expanded beyond its classical static framework. Contemporary economic research increasingly employs extended and modified versions of the model to analyze modern economic processes, including global value chains, environmental and energy issues, regional development, and sustainable growth. The integration of matrix theory into these extended models enhances their analytical power and allows for more accurate representation of real-world economic dynamics.

This article aims to explore the possibilities of the Leontief model based on matrix theory in the analysis of modern economic processes. Special attention is given to the mathematical structure of the model, its analytical capabilities, and its relevance in addressing current economic challenges. The study highlights the methodological significance of matrix-based approaches in economic analysis and demonstrates the continued relevance of the Leontief model in contemporary economic research.

Literature Review. The theoretical foundations of intersectoral economic analysis are closely associated with the development of input–output methodology, first systematically formulated by Wassily Leontief. In his seminal works, Leontief introduced a quantitative framework for representing the flows of goods and services between economic sectors through systems of linear equations. By expressing these relationships in matrix form, he demonstrated how the structure of production technologies determines the interdependence of industries and the overall balance of an economy. This approach laid the groundwork for the

application of matrix theory in economic modeling and earned Leontief the Nobel Prize in Economic Sciences [1].

Subsequent contributions by economists such as Paul Samuelson significantly advanced the mathematical interpretation of the Leontief model. Samuelson emphasized the importance of linear algebra in economic theory, particularly in analyzing equilibrium conditions and comparative statics. His work highlighted how matrix operations, including inversion and eigenvalue analysis, provide insights into the stability and responsiveness of economic systems modeled by input–output frameworks [2].

The dynamic aspects of economic growth and structural change were further explored by Robert Solow, whose research, while primarily focused on growth theory, contributed to understanding how intersectoral relationships evolve over time. Solow’s analytical perspective influenced later extensions of the Leontief model, especially those incorporating time-dependent coefficients and dynamic adjustments, thereby strengthening the connection between matrix-based models and real economic processes [3].

From a methodological standpoint, the integration of optimization and matrix theory into economic analysis was strongly influenced by Leonid Kantorovich. His work on linear programming and optimal resource allocation provided complementary tools for analyzing production systems and resource constraints. Kantorovich’s contributions reinforced the role of matrices not only in descriptive models like the Leontief framework but also in prescriptive economic analysis [4].

In the field of applied input–output analysis, comprehensive methodological developments were presented by Ronald E. Miller and Peter D. Blair, who systematically expanded the theoretical and empirical applications of the Leontief model. Their work demonstrated how matrix-based input–output models can be used to analyze regional economies, environmental impacts, and policy scenarios, thereby extending the relevance of the model to contemporary economic issues [5].

More recent studies have focused on adapting the Leontief model to modern economic challenges such as globalization, global value chains, and sustainable development. Researchers emphasize the importance of extended input–output tables and hybrid models that combine classical matrix formulations with environmental and energy data. These approaches illustrate that matrix theory remains a powerful analytical tool for capturing complex interdependencies in modern economies.

Overall, the reviewed literature confirms that the Leontief model, grounded in matrix theory, continues to be a central instrument in economic analysis. While its classical formulation provides a robust theoretical foundation, ongoing extensions and methodological refinements enable the model to address the complexities of contemporary economic systems. This body of research underscores the enduring relevance of matrix-based approaches in understanding and analyzing modern economic processes.

Main Part.

1. Matrix Representation of the Leontief Model

The Leontief input–output model provides a formal mathematical framework for describing intersectoral economic relationships through

matrix equations. Let the economy consist of n production sectors. The total output vector $x \in \mathbb{R}^n$ represents the gross production of each sector, while the final demand vector $y \in \mathbb{R}^n$ captures consumption, investment, and export demand. Intersectoral technological relationships are described by the matrix of technical coefficients

$$A = (a_{ij}),$$

where each element a_{ij} denotes the amount of input from sector i required to produce one unit of output in sector j .

The fundamental balance equation of the Leontief model is expressed as

$$\mathbf{x} = A\mathbf{x} + \mathbf{y}.$$

Rearranging this equation yields

$$(I - A)\mathbf{x} = \mathbf{y},$$

where I is the identity matrix. This formulation highlights the central role of matrix theory in the model, as the existence and properties of the solution vector x depend on the mathematical characteristics of the matrix $I-A$.

2. The Leontief Inverse and Economic Interpretation

A key concept in the matrix-based analysis of the model is the inverse matrix

$$(I - A)^{-1},$$

commonly referred to as the Leontief inverse. When this inverse exists, the solution to the system can be written as

$$\mathbf{x} = (I - A)^{-1}\mathbf{y}.$$

Economically, the Leontief inverse captures both direct and indirect input requirements across all sectors. Each element of the inverse matrix reflects the total increase in output of one sector necessary to satisfy a unit increase in final demand in another sector. This property allows researchers to analyze multiplier effects and to assess how shocks in demand propagate through the entire economic system.

The existence of the Leontief inverse is closely related to the spectral properties of the matrix A . In particular, if the spectral radius of A is less than one, the matrix $I-A$ is non-singular, ensuring a unique and economically meaningful solution. This condition is commonly interpreted as a requirement for technological feasibility and economic stability.

3. Stability and Structural Analysis Using Matrix Theory

Matrix theory provides powerful tools for investigating the stability and structural characteristics of the Leontief model. Eigenvalue analysis plays a crucial role in determining whether the modeled economic system can maintain equilibrium under changes in demand or production technology. If all eigenvalues of the technical coefficient matrix lie within the unit circle, the system exhibits stable behavior, meaning that production levels remain bounded and responsive to external changes.

Beyond stability analysis, matrix decomposition techniques enable the examination of structural changes in the economy. By comparing technical coefficient matrices across different periods, researchers can identify shifts in production technologies, changes in intersectoral dependencies, and trends in economic transformation. Such analyses are particularly relevant in the context of technological innovation and digitalization, where production structures evolve rapidly.

4. Applications to Modern Economic Processes

In contemporary economic research, the Leontief model based on matrix theory has been widely applied to analyze complex economic phenomena. At the national and regional levels, the model is used to evaluate the impact of policy interventions, investment projects, and demand shocks on sectoral output and employment. At the global level, extended input–output models capture international production networks and global value chains, allowing for the assessment of cross-border interdependencies.

Furthermore, matrix-based extensions of the Leontief model have been developed to address environmental and energy-related issues. By augmenting the technical coefficient matrix with environmental indicators, researchers can quantify the ecological footprint of economic activities and evaluate the sustainability of production structures. These applications demonstrate the flexibility of the matrix-theoretical framework in adapting the classical model to modern economic challenges.

5. Analytical Potential and Limitations

While the matrix-based Leontief model offers significant analytical advantages, it also has limitations that must be acknowledged. The classical model assumes fixed technical coefficients and linear relationships, which may not fully capture substitution effects and nonlinear dynamics present in real economies. Nevertheless, ongoing methodological developments, including dynamic and hybrid models, continue to enhance the analytical potential of the Leontief framework. Overall, the model originally proposed by Wassily Leontief, when interpreted through the lens of matrix theory, remains a powerful and

versatile tool for analyzing modern economic processes. Its ability to represent complex interdependencies in a rigorous mathematical form ensures its continued relevance in economic analysis and policy evaluation.

Conclusion

This study has examined the analytical possibilities of the Leontief input–output model when formulated within the framework of matrix theory and applied to the analysis of modern economic processes. The results confirm that the matrix-based representation of intersectoral relationships provides a rigorous and transparent method for describing the structural interdependencies of an economy. By expressing production, intermediate consumption, and final demand in matrix form, the model enables a systematic evaluation of equilibrium conditions and sectoral interactions.

The analysis demonstrates that key concepts of matrix theory, such as the Leontief inverse, eigenvalues, and stability conditions, play a central role in understanding the behavior of economic systems. In particular, the Leontief inverse allows for the identification of both direct and indirect effects of changes in final demand, thereby offering valuable insights into multiplier mechanisms and the transmission of economic shocks across sectors. These properties make the model especially useful for policy analysis and strategic economic planning.

Furthermore, the study highlights the relevance of the Leontief model in addressing contemporary economic challenges. Modern extensions of the model, grounded in matrix theory, have proven effective in analyzing global value chains, regional development patterns, and environmental–economic interactions. The adaptability of the matrix

framework enables the incorporation of additional dimensions, such as energy use and environmental impacts, enhancing the model's applicability to issues of sustainable development.

Despite its strengths, the classical Leontief model is subject to certain limitations, including the assumptions of fixed technical coefficients and linear production relationships. However, ongoing methodological advancements, such as dynamic and hybrid matrix-based models, continue to expand its analytical scope and realism. These developments ensure that the Leontief framework remains a relevant and powerful tool for modern economic analysis.

In conclusion, the Leontief model, when interpreted through the lens of matrix theory, retains significant theoretical and practical value in the study of contemporary economic processes. Its ability to capture complex intersectoral relationships in a mathematically coherent form makes it an essential instrument for both academic research and applied economic policy analysis.

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